Improvement of Active Microwave Device Modeling Using Filter-Bank Transforms

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***Abstract*— A new approach toward global modeling improvement of active microwave devices using filter-bank transforms is presented. A preconditioner based on these transforms is used to facilitate the iterative solution of the Poisson's equation. This equation must be solved in the excitation plane which the input voltage is applied, at each time step in the full-wave analysis of high-frequency active devices. The condition number of the preconditioned matrix and computational cost of the proposed method is better than the conventional (ILU) and wavelet-based preconditioners. This paper also presents a fundamental step toward applying filter-bank transforms to Maxwell's equations in conjunction with the hydrodynamic model in implicit schemes, aiming to decrease the simulation time of global modeling.**

I. Introduction

The full-wave analysis and simulation of high-frequency active devices and circuits (Global Modeling) should involve solving the equations that describe the electron transport physics in conjunction with Maxwell's equations to predict the interactions between the carriers and the propagating wave inside the devices [1]-[2]. This type of analysis involves a fair amount of advanced numerical techniques and different algorithms . As a result, its computational cost is very expensive [2]. Therefore, there is an imperative need to present a new approach to reduce the simulation time, while maintaining the same degree of accuracy presented by the global modeling techniques. A possible approach is to use multiresolution nonuniform grids that can be implemented using wavelets [3]-[4]. In conventional approach for implementing of global modeling using FDTD method, all the equations which include time derivative (hydrodynamic and Maxwell's equations) are represented by explicit FD schemes [1]. Only, solving the Poisson's equation (as an elliptic equation) leads to a large system of linear equations, $Ax = b$. Therefore, one of the important approaches for simulation time reduction of global modeling of active microwave devices is decreasing the solving time of the equation system, $Ax = b$, which obtained from the Poisson's equation.

Solving elliptic Partial Differential Equations (PDE's), or using implicit methods for solving time-dependent PDE's, results in large system of linear equations $Ax = b$. Size of the problems is often too large for using a direct solver, and one has to relay on iterative methods [5]. Such methods are dependent on the condition numbers of the operator matrices, A, in the sense that small condition numbers guarantee a fast convergence to the solution, whereas large condition numbers often imply that the convergence will be slow. For instance, solving the Poisson's equation on a large or nonuniform grid leads to a matrix with the large condition number. In this case, an effective preconditioning of the matrix A is usually required in order to keep the number of iterations small [5]. Using genetic-based algorithms that have been described and used in [6] is another approach for solving these problems. In [6] it has been shown that a genetic-based algorithm converges independent of the condition number of the matrix A which has been obtained from the Poisson's equation. Although genetic algorithms are unconditionally stable algorithms, but their convergence rate is very slow. Therefore, these algorithms cannot obviate the urgent need of the full-wave analysis of high frequency semiconductor devices that is simulation time reduction. Here, we propose to use an efficient method that not only guarantees obtaining the solution but also increases the speed of convergence.

It is important to note that in full-wave analysis of active devices, the Poisson's equation must always be solved in excitation plane which the input voltage is applied. For example, for the electromagnetic-wave analysis of MESFET transistor, an excitation voltage, $V_{gs}(t)$, is applied between the gate and the source electrodes at a plane (for instance, $z = 0$) [1]. This excitation is applied as a plane wave corresponds to the solution of the Poisson's equation of the applied voltage at each time step [1]. Then, the electric and magnetic fields are obtained in other sections by solving Maxwell's equations.

In this paper, a new proposed filter-bank based preconditioning method [7], is used to facilitate the iterative solution of the standard Poisson's equation which is an important and computationally expensive part of global modeling of active microwave devices. In [8], this method has been used to accelerate the iterative solutions of systems which coming from an implicit scheme of the Poisson's equation (modified Poisson's equation) and the carrier continuity equation.

Most of the orthogonal wavelet-based preconditioners provide effective schemes for matrices based on the structure of the matrices themselves rather than relying on detailed knowledge of the underlying problem from which they arise [9]. Here, we consider that the matrix, A, comes from the discretization of a PDE. This leads to better approximation of the transformed matrix with lower computational cost for preconditioning which can be reduced to $O(N)$ complexity [7]. Here, we use biorthogonal filter-bank transforms that have better performance than the conventional orthogonal filter-bank (wavelet) transforms, as will be shown.

Some well-known preconditioning methods such as incomplete LU factorization (ILU) and polynomial preconditioning method can be effective. However, they usually require well-

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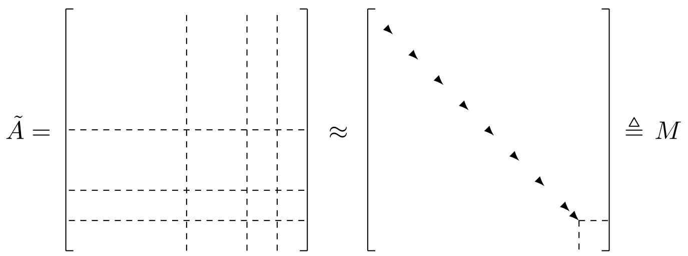

Fig. 1. Structure of matrix representation of transformed operator.

above $O(N)$ operations for implementation. Also, it will be shown that the performance of the filter-bank based preconditioning method is better than the ILU preconditioner.

II. Filter-Bank Based Preconditioner

By evolution of the wavelet techniques for preconditioning, an alternative approach has become available. Results show that general operators have a sparse representation in wavelet bases were derived in [10]. In [11] it was shown that the efficient decomposition level of the wavelet transform can be used to construct diagonal preconditioners when using a Galerkin method. The discrete analogue of the biorthogonal wavelet transforms relies on so called perfect reconstruction filter-bank transforms. In this paper we use biorthogonal filter-bank transforms and follow the preconditioner algorithm for discretized PDE's explained in [7]. Here we apply this algorithm to carry on the construction of preconditioner for the standard Poisson's equation, which is shown to improve the performance of the method. The motive for using orthogonal wavelet transforms is that the condition numbers of the transformed and untransformed operators are the same. However, the condition number actually decreases when doing biorthogonal filter-bank transforms.

A. Construction of the Preconditioner

If we consider a partial differential equation with special boundary conditions and assume that this problem is discretized with a finite difference method, a system of linear equations can be obtained as $Ax = b$. We want to use the filter-bank transform to precondition the operator. The idea is that the W parts of the transformed operator will be diagonally dominant, whereas the V is not, but this part is very small and can be directly inverted [7]. The filter-bank transform of matrix A is defined as:

$$\tilde{A} = TAT^T, \qquad (1)$$

where, matrix T performs $(n - n_0)$ steps in the filter-bank transform, $T^{n-n_0} : V^n \to (\oplus_{j=1}^{n-n_0} W^{n-j}) \oplus V^{n_0}$ (see [7] for notation and details). In general, the matrix representation of the transformed operator is shown in Fig. 1. Here, M is the approximation of the transformed operator. From its inverse that can be computed very easily, we will construct the preconditioner. Our approximation of $\tilde{A}$ is chosen like in Fig. 1 [7]:

$$M_{i,j} = \begin{cases} \tilde{A}_{i,j}, & i = j, i \in W^k, n_0 \le k < n, \\ \tilde{A}_{i,j}, & i, j \in V^{n_0}, \\ 0, & otherwise. \end{cases} \qquad (2)$$

Now, the filter bank preconditioning algorithm can be described as the following steps:

Step	Order of Complexity
Construction of M	$O(m^n)$
Construction of M^{-1}	$O(m^n + m_0^{3n})$
Multiplication, $M^{-1}T$	$O(m^n + m_0^{2n})$
Multiplication, $M^{-1}TAT^T$	$O(m^n + m_0^{2n})$
Total	$O(m^n + m_0^{3n})$

TABLE I

Arithmetic Complexity of Iterative Solver and Proposed Filter-Bank Preconditioning.

1) Take the filter-bank transform to get $\tilde{A} = TAT^T$.
2) Create the approximation of the transformed operator (M) by (2).
3) Use M^{-1} as preconditioner to solve $\tilde{A}\tilde{x} = \tilde{b}$, where $\tilde{b} = Tb$.
4) Apply backward filter-bank transform to $\tilde{x}$ to obtain $x = T^T\tilde{x}$.

According to the problem dimension, different filter-bank transforms can be used, *i.e.* for n-dimensional problems we can use the n-dimensional filter-bank transforms that are explained in detail in [7]. In the next section we use the tensor product filter-bank transform, called 2-D transform, for preconditioning the 2-D Poisson's equation. Also, we use Daubechies wavelets [10] for wavelet transforms and δ_1 and δ_3 filter-banks introduced in [7] for filter bank transforms.

B. Complexity Analysis of the Preconditioner

We now consider the computational cost of constructing the wavelet and filter-bank preconditioner for the preconditioning operation. This computational complexity which is shown in Table I, is a function of problem dimension, n, problem size, N (m points in each direction, $N = m^n$) and coarsest level (m_0 points in each direction, $N_0 = m_0^n$) [7]. For preconditioning, we altogether perform the transformation:

$$Ax = b \to (M^{-1}TAT^T)(Tx) = M^{-1}Tb. \qquad (3)$$

To summarize, in every step of constructing the proposed preconditioner and performing all computations for one iteration, the computational complexity and memory requirements are limited to be at most $O(m^n + m_0^{3n})$. The estimates are the same for different wavelet and filter-bank transforms. But, the constants are essentially proportional to the length of transform filters. It is interesting to note that if $m_0^{3n} \le m^n$, the computational complexity can be reduced to $O(N)$. This situation will appear in large problems when we transform the matrix to very low coarsest levels. Therefore, when the third power of the size of nondiagonal part of M is smaller than the size of A, the computational cost for preconditioning will be $O(N)$.

III. MESFET Simulation

To demonstrate the potential of the proposed approach, it is applied to a MESFET structure (Fig. 2) which is discretized by a uniform mesh of $65\Delta x \times 32\Delta y$. Dirichlet boundary conditions are used at the electrodes while Neumann boundary conditions are used at the other walls. The size of resulting Laplacian matrix will be equal to (2048×2048). The condition number of this matrix is 4051 and we apply the proposed filter-bank based and wavelet-based preconditioners to it to reduce this condition number. Fig. 3 demonstrates the two

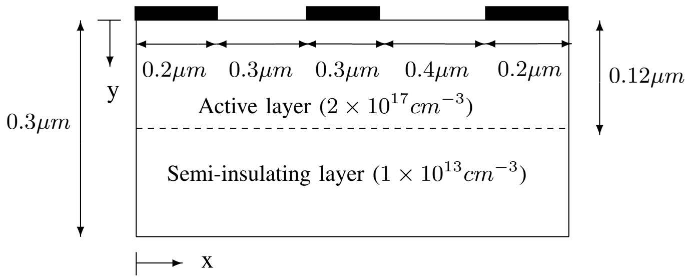

Fig. 2. Cross section of the simulated MESFET transistor.

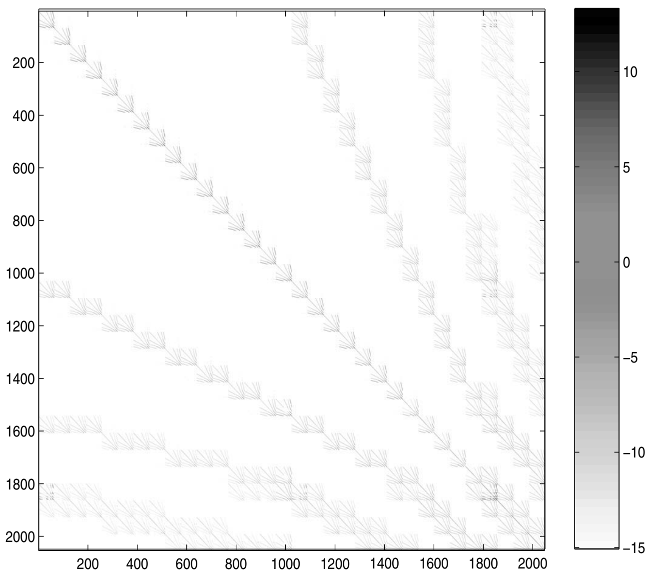

Fig. 3. Two-dimensional δ_1 filter-bank transform of the 2-D Laplacian operator matrix, drawn in decibel scale.

Type of filter-bank	Name	$\acute{M}$	Condition number
δ_1	$Delta1_{22}$	$(2^4.2^3) \times (2^4.2^3)$	592
δ_1	$Delta1_{32}$	$(2^3.2^3) \times (2^3.2^3)$	698
δ_1	$Delta1_{33}$	$(2^3.2^2) \times (2^3.2^2)$	820
δ_1	$Delta1_{43}$	$(2^2.2^2) \times (2^2.2^2)$	984
δ_3	$Delta3_{22}$	$(2^4.2^3) \times (2^4.2^3)$	774
δ_3	$Delta3_{32}$	$(2^3.2^3) \times (2^3.2^3)$	959
δ_3	$Delta3_{33}$	$(2^3.2^2) \times (2^3.2^2)$	1159
δ_3	$Delta3_{43}$	$(2^2.2^2) \times (2^2.2^2)$	1300

TABLE II

CONDITION NUMBER OF THE PRECONDITIONED LAPLACIAN OPERATOR MATRIX CORRESPONDING TO THE MESFET STRUCTURE FOR DIFFERENT FILTER-BANK TRANSFORMS.

Type of wavelet	Name	$\acute{M}$	Condition number
D_4	$Dab4_{22}$	$(2^4.2^3) \times (2^4.2^3)$	1140
D_4	$Dab4_{32}$	$(2^3.2^3) \times (2^3.2^3)$	844
D_4	$Dab4_{33}$	$(2^3.2^2) \times (2^3.2^2)$	810
D_4	$Dab4_{43}$	$(2^2.2^2) \times (2^2.2^2)$	790
$D_2(Haar)$	$Dab2_{22}$	$(2^4.2^3) \times (2^4.2^3)$	1940
$D_2(Haar)$	$Dab2_{32}$	$(2^3.2^3) \times (2^3.2^3)$	1412
$D_2(Harr)$	$Dab2_{33}$	$(2^3.2^2) \times (2^3.2^2)$	1381
$D_2(Harr)$	$Dab2_{43}$	$(2^2.2^2) \times (2^2.2^2)$	1250

TABLE III

CONDITION NUMBER OF THE PRECONDITIONED LAPLACIAN OPERATOR MATRIX CORRESPONDING TO THE MESFET STRUCTURE FOR DIFFERENT WAVELET TRANSFORMS.

dimensional δ_1 filter-bank transform of the obtained matrix. In Tables II and III, we present the variation of condition number of the preconditioned matrix according to the type of filter-bank and wavelet transforms and to the number of steps in the transform, which determines the size of nondiagonal part of matrix M called $\acute{M}$. We have used the tensor product (2-D) transforms for preconditioning. It is clearly shown that for the δ filter-bank transforms, the condition number decreases as the filter-bank is of lower order and as the decomposition levels decrease (Table II). But for the Daubechies wavelet transforms the condition number decrease as the wavelet is as higher order and as the decomposition levels increases (Table III). Fig. 4 shows the convergence behavior of the proposed preconditioner for different filter-bank and wavelet 2-D transforms. Convergence behavior of the preconditioned system is similar to the variation of its condition number. As it is seen, the convergence rate increases as the filter-bank is of lower order and as the decomposition levels decreases. But for the Daubechies wavelet transforms, the convergence rate increases as the wavelet is as higher order and as the decomposition level increases. We found that preconditioning using δ_1 filter-bank transform converges faster than the other filter-bank and wavelet transforms. By increasing the number of steps in the transform, the size of nondiagonal part of M decreases. Therefore, the computational complexity of the preconditioning method, which is equal to $O(N+\acute{M}^3)$ can be reduced by increasing the number of steps in the transform. It is interesting that we can obtain both good conditioning and low computational cost by using δ_1 filter-bank transform. For example, in $Delta1_{43}$ situation (Table II), the complexity is $O(2^{11} + (2^4)^3)$ which equals to $O(N)$, ($N = 2^{11}$ and $\acute{M} = 2^4$).

To compare the performance of the used preconditioner (filter-bank based preconditioner) with the well-known preconditioning methods, the convergence rate of the incomplete LU factorization (ILU) preconditioner which applied to our problem, has been illustrated in Fig. 4. As it is clearly seen, almost in all cases, for different filter-bank and wavelet transforms and decomposition levels, the convergence rate is faster than the ILU(0) preconditioner.

Now, we use the solution of the Poisson's equation obtained using the proposed filter-bank preconditioner for simulation of the considered MESFET transistor. The transistor model used in this study is a two-dimensional (2-D) simplified hydrodynamic transport model [12]. The semiconductor conservation equations are solved in conjunction with the Poisson's equation in excitation plane (cross section of the gate-source input voltage) and with the Maxwell's equations in other sections. An explicit finite-difference time-domain (FDTD) scheme was used to discretize each equation [13]. It is significant to indicate that the proposed algorithm gives precisely the same results obtained when the used iterative method does not employ the proposed preconditioner method. DC simulation results obtained from the 2-D simulation shows about 50% reduction in simulation time. In AC simulation, when Maxwell's equations together with the Poisson's equation (for excitation plane) must be solved, this reduction is about 20%.

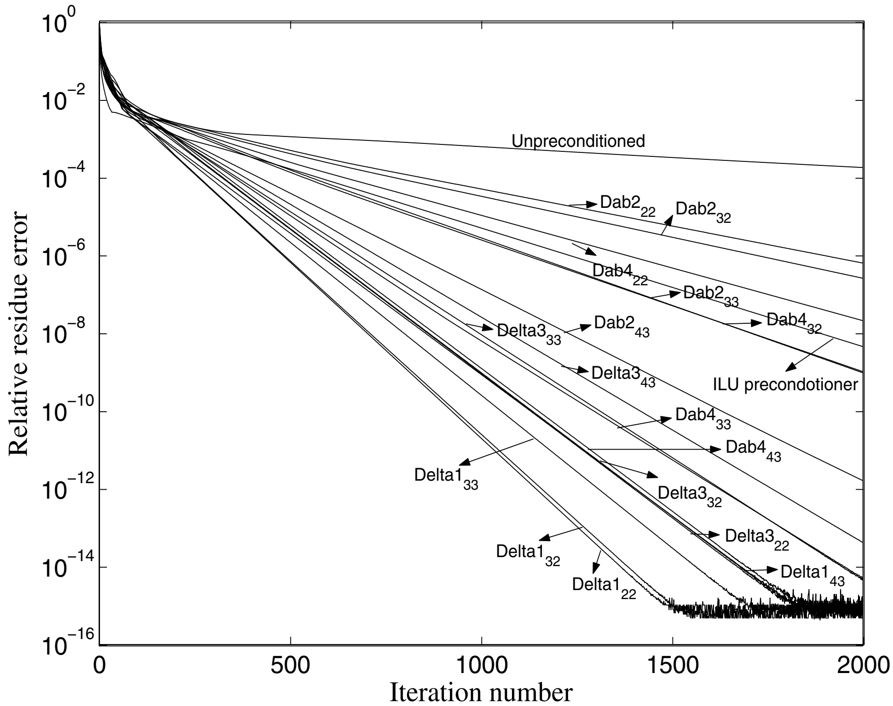

Fig. 4. Convergence behavior of the ILU preconditioner and the proposed preconditioner system by different wavelet and filter-bank transforms.

IV. CONCLUSION

In this paper, we have proposed to use a filter-bank based preconditioner for accelerating the iterative solution of the Poisson's equation in global modeling of active microwave devices. The convergence rate of the used preconditioning scheme, by different filter-bank and wavelet transforms, is faster than the well-known ILU method. Moreover, the computational cost of the considered method is as low as $O(N)$ which is very better than the other methods. It is because the scheme uses a very short filter bank and a good approximation of the transformed matrix with low computational cost to find its approximate inverse. This paper also presents a fundamental step toward applying filter bank transforms to Maxwell's equations in conjunction with the hydrodynamic model, aiming to decrease the simulation time of global modeling.

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