A Novel Approach to Radio Direction Finding and Detecting The Number of Sources Simultaneously: DMSAE Algorithm

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***Abstract*— In smart antenna systems, The localization and direction of desired source(s) must be known. In this paper, we introduce a method called "DMSAE" (Direction finding of Multiple Sources by Alternating Eliminations) to find the radio directions of signals impinging on an arbitrary antenna array and the number of the signals. This method which is based on the Minimum Variance Distortionless Response (MVDR) algorithm, can not only detect the direction and the number of sources, simultaneously, but also has better resolution ability compared to the other conventional direction finding algorithms. This method can also detect very weak sources when other powerful sources exist in the environment. In this method an iterative approach is employed for the detection and elimination of all sources by a null steering algorithm.**

I. INTRODUCTION

Localization of radiating sources by passive sensor arrays is one of the most important problems in Radar, Sonar, Radio-astronomy, and Seismology. The simplest problem in this context is the estimation of the direction-of-arrival (DOA) of narrow-band sources with the same known center frequency, which are located in the far field of an array composed of sensors with arbitrary locations and arbitrary directional characteristics [1]. In general, various DOA superresolution estimation methods are grouped in two classes, *Spectral Estimation Methods* and *Eigenstructure-based Methods* [1]. The first class, estimates the direction of arrival signal by computing the spatial spectrum and then determining the local maximums. The MVDR [2], the Maximum Likelihood [3], and the Linear Prediction [4] Methods are the examples in this class. One of the advantages of this class is the correct estimation of DOA's with no need to the number of sources as prior knowledge. But, they have low resolution to separate the close sources, and they cannot detect the existence and estimate DOA's of signals with high different power levels. This class has also the high sensitivity to the noise, array calibration, and array mismatching. The eigenstructure-based methods, a famous example is the MUSIC (**Mu**ltiple **Si**gnal **C**lassification) [5], are based on the fact that the space spanned by the eigenvectors of the array correlation matrix, may be partitioned in two subspaces, namely, the signal subspace and the noise subspace and the fact that the steering vectors corresponding to the directional sources are orthogonal to the noise subspace [5]. Simple formulation, high resolution, and lower sensitivity to the noise and array perturbations are advantages of these methods [1]. But, the number of sources as the input of algorithms must be known, and the performance of this class is dependent on the knowledge of this number. Moreover, some methods of this class, such as Root-MUSIC [6], ESPRIT [7], and Minimum-Norm [8], can only be applied to the arrays with special geometries, such as Uniform Linear Arrays (ULA) [1]. Detection of the number of sources impinging on the array in a passive sensor array system is the key step in the most of superresolution estimation techniques, especially in the eigenstructure-based methods [9]. The most commonly referred methods for detecting the number of sources do not have good performances at low SNR's and in practice, their noise assumptions are invalid [10].

In this paper we present a novel method to estimate simultaneously the number of the sources and find the DOA's of them. This method which combines The MVDR method and a null steering algorithm [11], iteratively, by eliminating the powerful estimated sources in previous steps, can detect and estimate the existence and location of weak and close sources. Using this method, we cover some disadvantages of the MVDR method and obtain a powerful approach in radio direction finding of sources with wide power levels and also in adaptive beamforming.

II. PROBLEM FORMULATION

Consider an array composed of p sensors with arbitrary locations and arbitrary directional characteristics, and assume that q narrow-band sources, centered around a known frequency, say ω_0, impinge on the array from directions $\theta_1, \theta_2, \ldots, \theta_q$. Since narrow-bandedness in the sensor array context means that the propagation delays of the signals across the array are much smaller than the reciprocal of the bandwidth of the signals, it follows that the complex envelopes of the signals received by the array can be expressed as

$$\mathbf{x}(t) = \sum_{k=1}^{q} \mathbf{a}(\theta_k) s_k(t) + \mathbf{n}(t). \tag{1}$$

where $\mathbf{x}(t)$ is the $p \times 1$ vectors

$$\mathbf{x}(t) = [x_1(t), \ldots, x_p(t)]^T \tag{2}$$

and $\mathbf{a}(\theta_k)$ is the *steering vector* or *array manifold* of the array toward direction θ_k.

$$\mathbf{a}(\theta_k) = [a_1(\theta_k)e^{-j\omega_0\tau_1(\theta_k)}, \ldots, a_p(\theta_k)e^{-j\omega_0\tau_p(\theta_k)}]^T. \tag{3}$$

Here, T denotes the transpose. The vector of the received signals can be expressed more compactly as

$$\mathbf{x}(t) = \mathbf{A}(\mathbf{\Theta})\mathbf{s}(t) + \mathbf{n}(t), \tag{4}$$

where $\mathbf{A}(\mathbf{\Theta})$ is the $p \times q$ matrix of the steering vectors

$$\mathbf{A}(\mathbf{\Theta}) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_q)], \tag{5}$$

$\mathbf{s}(t)$ is the $q \times 1$ vector of the signals

$$\mathbf{s}(t) = [s_1(t), \ldots, s_q(t)]^T, \tag{6}$$

and $\mathbf{n}(t)$ is the $p \times 1$ vector of additive noise in sensors

$$\mathbf{n}(t) = [n_1(t), \ldots, n_p(t)]^T. \tag{7}$$

Generally, array manifold ($\mathbf{a}(\theta)$) can be a function of two variables θ and ϕ ($\mathbf{a}(\theta, \phi)$) corresponding to azimuth and elevation angles, respectively. The Direction finding problem is to estimate $(\theta_1, \phi_1), \ldots, (\theta_q, \phi_q)$ the directions of the sources from the M samples (*snapshots*) of the array outputs $\mathbf{x}(t_1), ..., \mathbf{x}(t_M)$.

III. MVDR AND LCMV METHODS

The **M**inimum **V**ariance **D**istortionless **R**esponse (MVDR) algorithm that is commonly referred to as Capon's method, in fact, is a beamformer used in adaptive array systems [9]. In a minimum variance distortionless response beamformer, the array weights are chosen to pass the desired direction (look direction) signal without any distortion and to reject maximally the interfering signals and white noise.

We define the array covariance matrix as:

$$\mathbf{R} \equiv E[\mathbf{x}(t)\mathbf{x}^\dagger(t)] = \mathbf{A}\mathbf{S}_E\mathbf{A}^\dagger + \sigma^2\mathbf{I}, \tag{8}$$

where $E[.]$ denotes expectation operators, $\dagger$ denotes Hermitian transpose (conjugate transpose), $\mathbf{I}$ is the $p \times p$ identity matrix, and $\mathbf{S}_E = E[\mathbf{s}(t)\mathbf{s}^\dagger(t)]$ is the $q \times q$ matrix of signal amplitudes.

Let $\mathbf{a}(\theta_1)$ represents the desired signal direction (look-direction) vector. The signals received at the sensors are combined to form the array output

$$y(t) = \boldsymbol{w}^\dagger\mathbf{x}(t) = \boldsymbol{w}^\dagger\mathbf{A}(\Theta)\mathbf{s}(t) + \boldsymbol{w}^\dagger\mathbf{n}(t), \tag{9}$$

where $\boldsymbol{w}$, the weight vector in the MVDR beamformer, is determined to minimize the output power under the criterion that the gain in the look-direction is unity, *i.e.*,

$$\begin{array}{ll} minimize & \boldsymbol{w}^\dagger\mathbf{R}\boldsymbol{w} \\ subject\ \ to & \boldsymbol{w}^\dagger\mathbf{a}(\theta_1) = 1. \end{array} \tag{10}$$

Equation (10) has an analytical solution [12]

$$\boldsymbol{w}_{mvdr} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_1)}{\mathbf{a}^\dagger(\theta_1)\mathbf{R}^{-1}\mathbf{a}(\theta_1)}. \tag{11}$$

Using this weight vector, the system output power is:

$$P_{mvdr}(\theta_1) = y(t)y^\dagger(t) = \boldsymbol{w}^\dagger\mathbf{x}(t)\mathbf{x}^\dagger(t)\boldsymbol{w} = \frac{1}{\mathbf{a}^\dagger(\theta_1)\mathbf{R}^{-1}\mathbf{a}(\theta_1)}. \tag{12}$$

In finite data case, an estimate of the array covariance matrix is defined as

$$\mathbf{R}_s = \frac{1}{M}\sum_{i=1}^{M}\mathbf{x}(t_i)\mathbf{x}^\dagger(t_i), \tag{13}$$

where M is the number of snapshots. Thus in finite data case, $\mathbf{R}$ in the above equations replaces with $\mathbf{R}_s$.

To use the MVDR algorithm for the parameter estimation and direction finding, we plot $P(\theta)$ as a function of the look-direction (θ_1). The location of its q peaks ($\hat{\theta}_1 \ldots \hat{\theta}_q$) correspond to the direction of sources. In the MVDR method, the minimum variance distortionless beamformer is derived by imposing a linear constraint, $\boldsymbol{\omega}^\dagger\mathbf{a}(\theta_1) = 1$. Another beamformer, called, the **L**inear **C**onstrained **M**inimum **V**ariance (LCMV) method has been developed in which additional linear constraints are imposed to make the beamformer more robust [13]. A set of linear constraints, by the $q \times (m+1)$ matrix, $\mathbf{C}_m$, whose columns are linearly independent are defined [13]. We require that

$$\boldsymbol{w}^\dagger\mathbf{C}_m = \mathbf{g}^\dagger. \tag{14}$$

Subject to the constraint in (14) the LCMV beamformer performs the following optimization

$$minimize \quad \mathbf{w}^\dagger\mathbf{R}\boldsymbol{w}. \tag{15}$$

In order to have unity gain in θ_1 direction and reject the sources in $\theta_{Null\_1}, \ldots, \theta_{Null\_m}$ directions (*null steering algorithm*), the constraint matrix and $\mathbf{g}^\dagger$ vector must be

$$\begin{array}{rcl} \mathbf{C}_m & = & [\mathbf{a}(\theta_1), \mathbf{a}(\theta_{Null\_1}), \ldots, \mathbf{a}(\theta_{Null\_m})] \\ \mathbf{g}^\dagger & = & [1, 0, \ldots, 0]^T. \end{array} \tag{16}$$

The solution of minimization of (15) subject to the constraint in (16) is [13]:

$$\boldsymbol{w}_{lcmv}^\dagger = \mathbf{g}^\dagger\ [\mathbf{C}_m^\dagger\ \mathbf{R}^{-1}\mathbf{C}_m]^{-1}\mathbf{C}_m^\dagger\ \mathbf{R}^{-1}. \tag{17}$$

Note that $(m+1)$ must be less than the number of sensors. Using this weight vector and changing the look-direction (θ_1 to θ), the spatial spectral estimation function will be

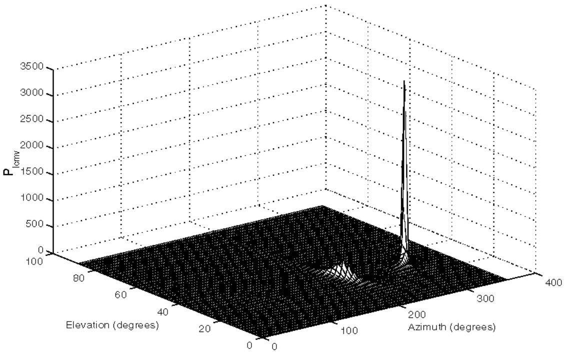
$$\begin{array}{rcl} P_{lcmv}(\theta) & = & y(t)y^\dagger(t) = \boldsymbol{w}_{lcmv}^\dagger(\theta)\mathbf{x}(t)\mathbf{x}^\dagger(t)\boldsymbol{w}_{lcmv}(\theta) \\ & = & \boldsymbol{w}_{lcmv}^\dagger(\theta)\ \mathbf{R}\ \boldsymbol{w}_{lcmv}(\theta). \end{array} \tag{18}$$

IV. DMSAE METHOD

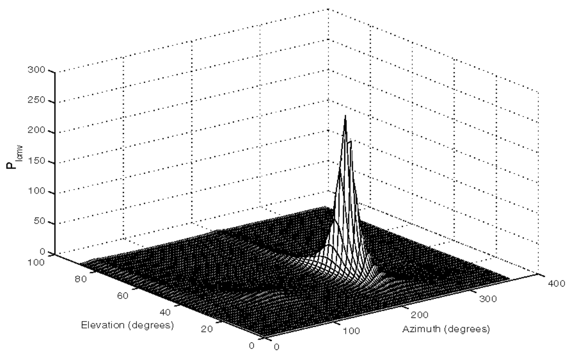
The DMSAE (**D**irection finding of **M**ultiple **S**ources by **A**lternating **E**liminations) algorithm is based on the LCMV method for radio direction finding. The algorithm operates iteratively by estimating the direction of arrival of one signal from the directional sensitivity pattern of the array whilst suppressing all other signal estimates through the null steering by the LCMV method.

Initially, a single signal component is assumed. When the estimated directions of all the signals remain stable and there is still significant energy being detected, the number of signals searched for is increased by one. The search terminates when the spectral estimation function peak is less than a predefined value or when a new estimate is identical to one of the nulled directions. The latter situation arises at low residual energy levels when noise and system calibration errors result in a phase distribution across the array which does not exactly match that expected for any direction of arrival (i.e. the vector required to null out this residual does not lie within the array manifold). The algorithm operates as the following:

1) The largest peak in the spatial spectral estimation function (Eq. (12)) is found, and the threshold is set according to the system noise level and system calibration accuracy. The number of sources is assumed to be 1 ($no. = 1$) related to the largest peak and its angle is called θ_{Null_1}. $n = 1$ (first iteration).



(a)



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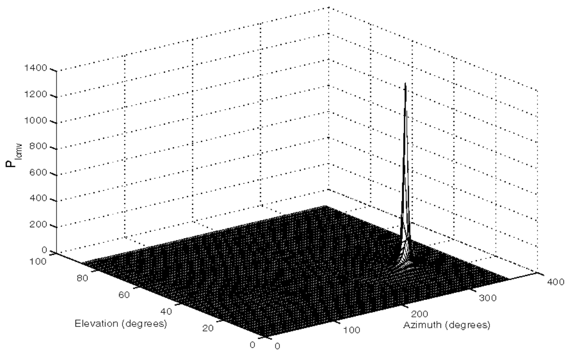
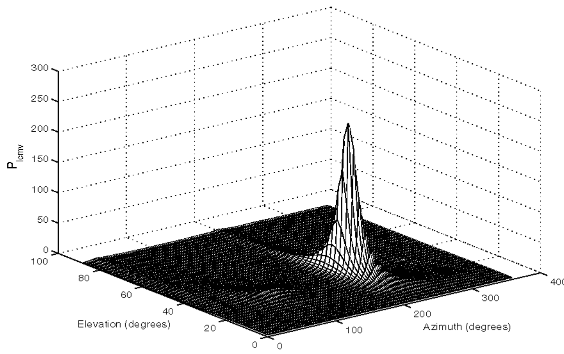


(c)



(d)

Fig. 1. $P_{lcmv}(\theta, \phi)$ function at sequential iterations of DMSAE method: (a)-(d) corresponding to iterations 1-4

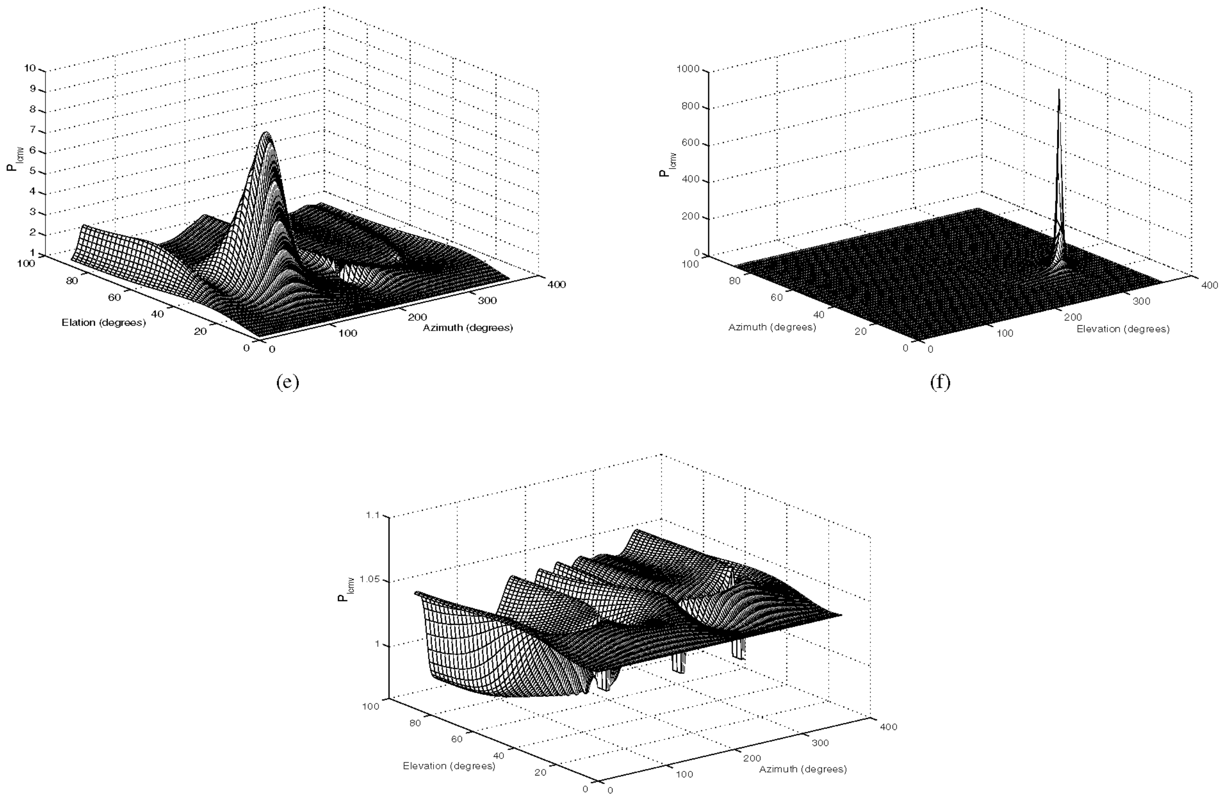
2) The weight vector of the LCMV method is formed from $\mathbf{C}_n$ (Eq. (16)). ($\mathbf{C}_n$ is calculated using the steering vectors θ_{Null_n} to $\theta_{Null\_(n-(no.-1))}$)
3) The spatial spectral estimation function of the LCMV method (Eq. (18)) is calculated, and the largest peak of it ($\theta_{Null\_(n+1)}$) is found.
4) If the amplitude of the peak from step (3) is less than the threshold or angle $\theta_{Null\_(n+1)}$ is equal to one of the rejected angles (θ_{Null_n} to $\theta_{Null\_(n-(no.-1))}$), then all the present signals have been found and the algorithm is terminated, END.
5) If $\theta_{Null\_(n+1)} = \theta_{Null\_(n-no.)}$ then $no. = no. + 1$.
6) $n = n + 1$, go to step (2).

Now, as an example, consider a scenario with three sources of unequal amplitude form DOA's of $\begin{pmatrix} \theta_1 = 100^\circ \\ \phi_1 = 30^\circ \end{pmatrix}$, $\begin{pmatrix} \theta_2 = 210^\circ \\ \phi_1 = 30^\circ \end{pmatrix}$ and $\begin{pmatrix} \theta_3 = 300^\circ \\ \phi_3 = 30^\circ \end{pmatrix}$. The sources SNR's are different; $SNR_1 = 5\ dB, SNR_2 = 20\ dB$ and $SNR_3 = 30\ dB$. This means that the power of source 3 is about 300 times the power of source 1 and for source 2 is about 30 times that for source 1. Applying the MVDR algorithm directly to this scenario only two sources, can be detected, due to high difference between powers of sources. Now we apply the DMSAE algorithm to this scenario and its performance will be demonstrated. The result of applying the DMSAE algorithm to this scenario is shown in Table 1 and Fig. 2. The first column of this table shows the iteration number, the second column is the amplitude of the largest peak in the spectrum function and the corresponding DOA is in the third column. As illustrated in Fig. 2-a, at the first iteration ($n = 1$), the azimuth angle of the most powerful source is 302° and its elevation angle is 32° corresponding to source 3. In this figure which is related to the output of the MVDR method, only two sources have been appeared. At the second iteration, by eliminating this source, The location of maximum is transferred to $\begin{pmatrix} \theta_3 = 212^\circ \\ \phi_3 = 32^\circ \end{pmatrix}$, and the third source which was hidden, appears (Fig. 2-b). Now we can eliminate the two detected sources and estimate the precise location of other sources. But in order to confirm the previous estimations, at the third iteration, we only eliminate the latest maximum. The largest peak obtained in this step is $\begin{pmatrix} \theta_3 = 300^\circ \\ \phi_3 = 32^\circ \end{pmatrix}$. If the DOA's of sources at iteration 1 and iteration 3 are equal, we consider this location as the correct DOA related to one

Iteration (n)	Largest peak	DOA
1	3489	$(302^\circ, 32^\circ)$
2	268	$(212^\circ, 32^\circ)$
3	1370	$(300^\circ, 32^\circ)$
4	250	$(212^\circ, 32^\circ)$
5	9	$(102^\circ, 32^\circ)$
6	972	$(300^\circ, 32^\circ)$
7	1.04	$(332^\circ, 66^\circ)$

TABLE I

DOA OF THE LARGEST PEAK AND ITS AMPLITUDE AT SEQUENTIAL ITERATIONS OF DMSAE METHOD



(e) (f) (g)

Fig. 1. *(Cont.)* (e)-(g) corresponding to iterations 5-7

source and eliminate two sources at the following iterations. Now, in order to find the accurate location of sources, we only eliminate the latest maximum again, at iteration 4. In this step, the location of the largest peak is equal to the location of the largest peak at iteration 2. Therefore, we eliminate the two sources corresponding to the latest two iterations (iteration 4 and 3) at iteration 5. The DOA of the third source, which did not appear in the MVDR method, can be estimated after canceling the two powerful sources, at iteration 5 (Fig. 2-e). This sequence is continued. Because the DOA of the largest peak at iteration 6 is equal to it at iteration 3, we eliminate the three sources at iteration 7. As shown in Table 1, the amplitude of the peak in this step is 1.04 which is less than other peaks. The threshold of this scenario is equal to 2; and because the amplitude of the peak at iteration 7 is less than it, the algorithm is terminated. It is clear, that the number of sources is 3 and the locations of them are the DOA's at iterations 4, 5 and 6.

V. Conclusion

We have presented a novel and efficient method, referred to as the DMSAE method, for radio direction finding. This method not only does not need the number of sources as a prior knowledge for the DOA estimation, but also, can estimate the number of sources accurately. In this algorithm an iterative method is employed for the detection and elimination of all sources by a null steering algorithm. The resolution of this algorithm is more than the spectral estimation methods and can also detect very weak sources in the presence of other powerful sources in the environment.

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