Improved Meshless Method Using Direct Shape Function for Computational Electromagnetics

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***Abstract* — In this paper, an improved meshless method is presented by eliminating the time consuming process of determining the shape function. We introduce a direct approach, based on the properties, which is required for any shape function, to prepare the shape function and propose a new meshless method. The proposed method does not need to any predetermined basis or weighting functions. Another advantage of the introduced method is its capability for applying desirable changes to the shape function. It is seen that the method has good accuracy and is faster than the conventional meshless methods. Its application in an electromagnetic field reflection computation is also presented.**

***Index Terms* — Computational electromagnetics, meshless method, shape function, electrostatic analysis.**

I. INTRODUCTION

With the advent of computational technologies, many engineering problems can be solved with numerical methods such as finite-element method (FEM) [1] and finite-difference method (FDM). However, these methods are somewhat limited. For instance, FDM uses a grid (structured mesh), that does not fit well to complex geometries such as those with curvatures or with very different geometric feature sizes. These characteristics may lead to discretization errors or to meshes that are too refined because of small features in the geometric input.

FEM has been most widely used as it can handle complicated geometry with the help of a mesh generation program. However, computation accuracy of FEM depends on the quality of its mesh. Despite considerable effort has been devoted to improve the design of the mesh and the algorithm to generate it, the creation of a proper element structure has remained a challenge; human involvement is still unavoidable for most of engineering analysis with FEM.

Meshless methods, developed in the last decade, form a new class of numerical techniques which their main objective is to overcome the limitations imposed by traditional mesh structured methods [2]. The mesh-free characteristic makes meshless methods very useful, especially for modeling discontinuities and moving boundaries where the complexity in meshing and re-meshing of entire problem structure is eliminated. Meshless methods in seeking for an approximate solution to a problem governed by PDEs and boundary conditions, first needs to approximate the unknown field function using trial or shape functions. There are various techniques for meshless shape function constructions which can be categorized into three classes; radial point interpolation method (RPIM), weighted-least square method (WLS) and polynomial point interpolation method (PIM) [3]. All of these methods use special basis functions; for example, RPIM uses radial basis functions (RBF), such as multiquadric function (RPIM-MQ) [4], Gaussian function, thin plate spline function, and logarithmic function. Using basis function, finally, leads to produce a special function with specific characteristics corresponding to specifications of its basis function. From this stage on, the method simply uses this function called shape function.

Since construction of a shape function needs to compute a matrix inversion which usually is a time consuming process, the capability of the conventional meshless methods is limited by their relatively high computational complexity. This issue, especially, will appear when problems with large structures are simulated. An alternative approach to solve this problem is eliminating this stage and directly achieving a shape function without damaging the framework of the method.

In this paper, we introduce a general function which satisfy the necessary conditions for shape functions and can be used as a direct shape function for meshless methods without using any basis functions. The particular capabilities of the proposed function would be investigated and its efficiency will be compared with a conventional scheme of meshless method.

II. IMPROVED MESHLESS METHOD

As it was mentioned, conventional meshless methods need to compute the inversion of a matrix, which is usually an expensive process, to obtain the shape functions. In [5], for the first time, the authors introduced a general function which can be used as a shape function for meshless methods, directly.

This direct shape function satisfies the necessary conditions for some of the shape functions, e.g., delta Kronecker, continuity and partition of unity conditions and can be constructed faster without using any basis functions. It has been illustrated that the meshless method based on this shape function works well with scattered points and has better accuracy in uniform and non-uniform node distributions in electrostatic problems [5].

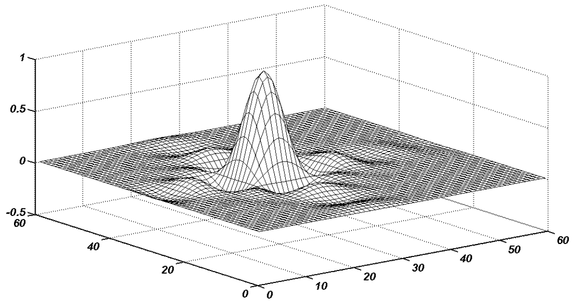

Fig. 1. RPIM-MQ shape function

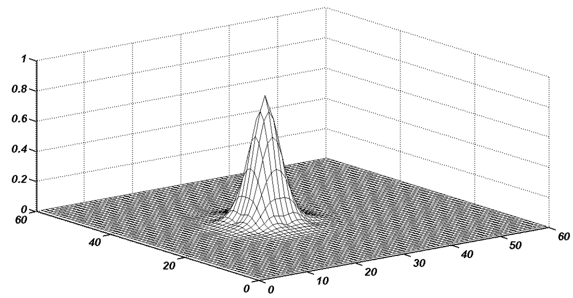

Fig. 2. Proposed shape function

But, because of satisfying the Kronecker condition, the function structure is nearly complex and the computational time needed to construct it is roughly high. In this paper for more reduction of the computational time, a simple function as a shape function is introduced that can be constructed, rapidly. This proposed shape function has partition of unity property and high order of continuity.

A. Representation of direct shape function

Here, according to the necessary conditions that a shape function must be having, we introduce a new shape function directly without using any basis functions and superfluous time consuming process. This function has also the capability to adapt itself in special cases and increase the accuracy and the pace of the meshless method. Following function has continuity and partition of unity conditions, which are essential provisions of any shape function:

$$\varphi_i(\mathbf{x}) = \frac{\exp(-\alpha\,|\,\mathbf{x}-\mathbf{x_i}\,|)}{N(\mathbf{x})} \qquad (1)$$

where $\mathbf{x}$ is the vector of space coordinates $\mathbf{x}^T = [x \;\; y]$ (for two-dimensional problems) and $\mathbf{x_i}$ is the space coordinate of nodes. α is an independent coefficient which can change the overhang width of the shape function and its optimal setting increases the accuracy of the method. N(x) is a function which can force the partition of unity condition, i.e.,

$$\sum_{i=1}^{n} \varphi_i(\mathbf{x}) = 1 \qquad (2)$$

where M is the total number of nodes. At first, it is not needed to have the exact formula for N(x), but in programming we can easily specify it by summing all shape functions ($\varphi_i(\mathbf{x})$) without considering N(x) and dividing all of them by total. This shape function, like some of the conventional shape functions, do not satisfy the Kronecker delta function property, i.e.,

$$\varphi_i(\mathbf{x} = \mathbf{x_i}) = \begin{cases} 1 & \mathbf{x} = \mathbf{x_i} \\ 0 & other \end{cases} \qquad (3)$$

Therefore, in the meshless method which this direct shape function is used, like MLS method, special treatments are needed to enforce the essential boundary conditions [6]. It is interesting to note that in this shape function, by getting smaller the overhang radius of the function using correct set of α, the value of the shape function in the other nodes would be close to zero. So, the boundary conditions will be enforced accurately with no trouble [7], [8]. The derivatives of these shape functions can be obtained simply in the closed form (not numerically), which is necessary for a truly meshless method.

Simplicity of the proposed function leads to fast obtaining of all shape functions and decrease the simulation time of the meshless method. Now, an example is presented to illustrate the shape of the conventional RPIM shape function and the introduced function. Figs. 1 and 2 show the shape of the RPIM-MQ shape function and the proposed shape function using 100 nodes in a rectangular domain, respectively. These (10×10) nodes are regularly and evenly distributed in the support domain. In these figures, a total of (60×60) points are used to evaluate and plot the shape functions.

B. Boundary-value problem

In almost all electromagnetic problems, the boundary-value problem under consideration is defined by the second order differential equation [1]

$$\frac{\partial}{\partial x}(\alpha_x \frac{\partial U(\mathbf{x})}{\partial x}) + \frac{\partial}{\partial y}(\alpha_y \frac{\partial U(\mathbf{x})}{\partial y}) + \beta U(\mathbf{x}) = f \qquad (4)$$

where U(x) is the unknown field function, αx, αy and β are known parameters or functions associated with the physical properties of the solution domain, and f is a known source or excitation function. The ordinary two-dimensional Laplace equation is a special case of (4). The boundary conditions to be considered are given by

$$U = P \qquad on \qquad \Gamma_1 \qquad (5)$$

and

$$(\alpha_x \frac{\partial U}{\partial x}\hat{x} + \alpha_y \frac{\partial U}{\partial y}\hat{y}).\hat{n} + \gamma U = q \qquad on \quad \Gamma_2 \qquad (6)$$

where Γ (= $\Gamma_1 + \Gamma_2$) denotes the contour or enclosing the area Ω (problem domain), $\hat{n}$ is its outward normal unit vector, and γ, p and q are known parameters.

C. Variational formulation

The variational problem equivalent to boundary value problem above is given by [1]

$$\begin{cases} \delta F = 0 \\ U = P \qquad on \qquad \Gamma_1 \end{cases} \qquad (7)$$

where

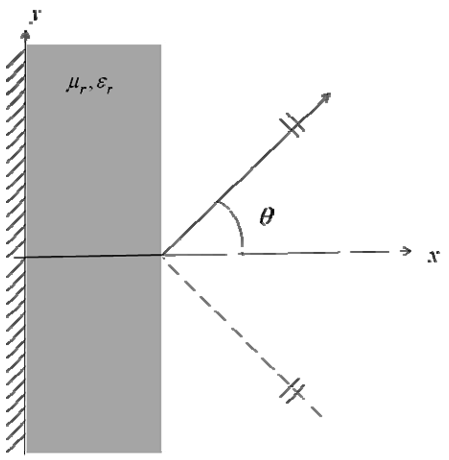

Fig. 3. Plane wave reflected by a metal-backed dielectric slab.

$$F(U) = \frac{1}{2}\iint_{\Omega}\left[(\alpha_x \frac{\partial U}{\partial x})^2 + (\alpha_y \frac{\partial U}{\partial y})^2 + \beta U^2\right] d\Omega + \int_{\Gamma_2} (\frac{\gamma}{2}U^2 - qU)\, d\Gamma - \iint_{\Omega} fU \, d\Omega \quad (8)$$

The field function (U(x)) would be approximated in the meshless method as:

$$\widetilde{U}(\mathbf{x}) = \sum_{i=1}^{n} \varphi_i(\mathbf{x}).a_i \quad (9)$$

where a_i are the unknown nodal function values. To find a_i, substitute (9) into (8), take the derivative of F with respect to a_i and it is set equal to zero to enforce condition (7). So, obtain (for simplicity assume γ = q = 0)

$$\frac{\partial F}{\partial a_i} = \sum_{j=1}^{n} a_i \iint_{\Omega} (\alpha_x \frac{\partial \varphi_i}{\partial x}.\frac{\partial \varphi_j}{\partial x} + \alpha_y \frac{\partial \varphi_i}{\partial y}.\frac{\partial \varphi_j}{\partial y} + \beta \varphi_i \varphi_j) d\Omega - \iint_{\Omega} f\varphi_i \, d\Omega \quad (10)$$

III. NUMERICAL RESULT

In order to evaluate the efficiency and accuracy of the proposed shape function for meshless method, an electromagnetic problem with analytical solution is analyzed here. The problem under consideration has been illustrated in Fig 3, where a uniform plane wave is incident upon an inhomogeneous dielectric slab backed by a conducting plane. The dielectric slab has thickness L, relative permittivity $\varepsilon_r = 4 + (2 - 0.1j)(1 - x/L)$, and permeability $\mu_r = (2 - 0.1j)$. The surrounding medium is free space having $\varepsilon_r = \mu_r = 1$. We are interested in finding the power reflected by the slab.

It is well known that any plane wave can be decomposed into an E_z-polarized plane wave having only a z-component for the electric field and a H_z-polarized plane wave having only a *z*-component for the magnetic field [1]. Therefore, it is sufficient to consider these two polarization cases only. For the E_z-polarization case, the incident wave can be represented as:

$$E_z^{inc}(x, y) = E_0 e^{jk_0 x\cos\theta - jk_0 y \sin\theta} \quad (11)$$

where is E_0 a constant denoting the magnitude of the incident field and θ is the angle of incidence defined in Fig. 3. Obviously, to satisfy the field continuity condition at the interfaces perpendicular to the x-axis, the total field must have a common factor $e^{-jk_0 y \sin\theta}$. With this observation, the scalar Helmholtz equation governing the electric field E_z reduces to

$$\frac{d}{dx}(\frac{1}{\mu_r}\frac{dE_z}{dx}) + k_0^2(\varepsilon_r - \frac{1}{\mu_r}\sin^2\theta)E_z = 0 \quad (12)$$

The boundary condition to be imposed on E_z is the Dirichlet boundary condition, i.e.,

$$E_z\,|_{x=0} = 0 \quad (13)$$

For the H_z-polarization case, related equations can be obtained, similarly.

This problem can be solved analytically by first dividing the slab into many thin layers. Within each layer the relative permittivity and permeability can be approximated as constants. By imposing the boundary conditions at the interface between the two layers, the reflection coefficient on the interface of each layer and also the interface of the slab can be obtained [1].

Now, let us consider conventional and proposed meshless method solutions to this problem. The domain of the problem obviously semi-infinite (0 ⩽ x < ∞), but the meshless methods cannot be applied directly to such an unbounded problem. Therefore, we reduce the domain by introducing an artificial boundary with an appropriate absorbing boundary condition as [1]:

$$[\frac{dE_z}{dx} + jk_0\cos\theta.E_z(x)]_{x=0} = 2jk_0\cos\theta.E_0.e^{jk_0 d\cos\theta} \quad (14)$$

where d is the position of this artificial boundary. By comparing (12) with (4), and (14) with (6), we recognize that for the E_z-polarization case,

$$U = E_z \quad , \quad \alpha_x = \frac{1}{\mu_r} \quad , \quad \alpha_y = 0 \quad , \quad \beta = k_0^2(\varepsilon_r - \frac{1}{\mu_r}\sin^2\theta) \quad (15)$$

$$\gamma = jk_0\cos\theta.E_z(x) \quad , \quad q = 2jk_0\cos\theta.E_0 e^{jk_0 d\cos\theta} \quad (16)$$

Assume the dielectric slab is 2.5 free-space wavelengths thick (L = 2.5λ0) and the absorbing boundary is considered at *2L* (*d = 2L*). For numerical implementation of the meshless method using the direct shape function, the solution domain is discretized by a one-dimensional uniform grid of nodes (200 nodes). The shape parameter, α, of the proposed shape function is considered as α = 0.7d_c, where dc is the average nodal spacing in the meshless method [3]. The visibility method [9] is used to build the interface condition. This technique implies that only the nodes that are in the same medium as the integration point x_q can belong to its influence domain. By this way, the discontinuity in the shape function is imposed.

The results are shown in Fig. 4. In this figure, the reflection coefficients for different values of θ in E_z-polarization case are illustrated. Two sets of the meshless method results are

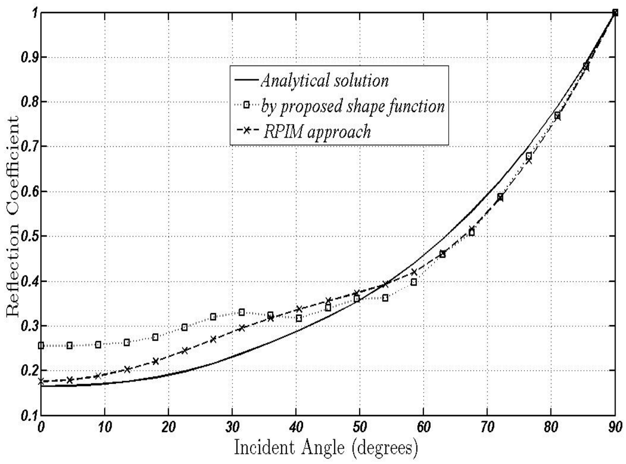

Fig. 4. Reflected coefficient of the metal-backed de electric slab.

presented and compared with the exact (analytical) solution, one with the proposed meshless method and the other with the conventional RPIM method (based on MQ-RBFs) with the same nodes. As shown, there is a good agreement between the proposed meshless method solution and the exact one. Although, the accuracy of the RPIM method becomes a little better especially for a small angle of incidence in this situation, the proposed method solution approaches the exact solution as the number of nodes increases.

As it was mentioned, one of the most advantages of the introduced shape function is its low computational complexity. As seen in Fig. 5, when the number of nodes, $M(=N^2)$ increases, the RPIM method processing time increases, fast. This is due to the computation of a $(M \times M)$ matrix inversion for its shape functions construction. But by using the direct shape function, there is no need to compute this matrix inversion. So, in the RPIM, by increasing the number of nodes when the dimension of the resulted matrix is enlarged, the load of calculations will increase, extremely.

It is interesting to note that in some problems which there is the rapid variations of parameters, not only using the direct shape function leads to reduction of the simulation time but also the accuracy would enhance in comparison with the most of the conventional meshless methods [5].

IV. CONCLUSION

In this work, we introduced a new approach to increasing the performance of the meshless methods by constructing the shape functions, directly. The proposed shape function can be constructed easily and there is no need to calculate a matrix inversion which can lead to the simulation time reduction. Some numerical results for an electromagnetic problem were presented. The results showed, in the conventional meshless meshless methods unlike the proposed method, when the problem matrix is enlarged to higher dimensions, the load of calculations would increase, extremely.

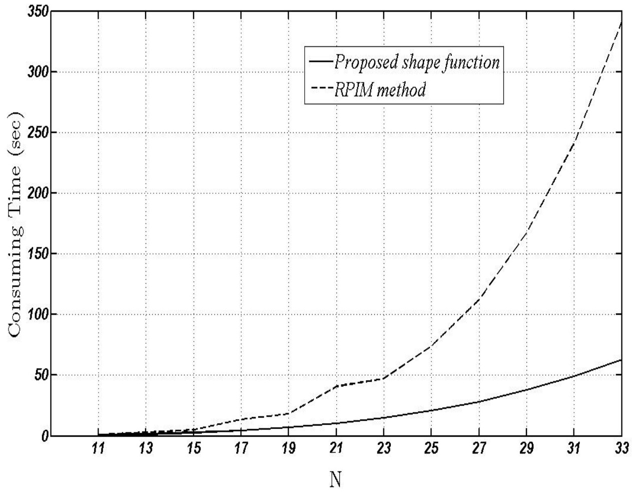

Fig. 5. Consuming time to construct the shape functions in direct shape function and RPIM-MQ approach for different number of nodes.

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