Time Domain Analysis of Lossy Nonuniform Transmission Line Using FDTD Technique

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Abstract - In this paper, an efficient numerical method for transient analysis of lossy nonuniform transmission lines is presented. This method considers the frequency-dependent conductor losses into the time-domain solution of the nonuniform multiconductor transmission lines. The results of this method are compared with the conventional time-domain to frequency-domain (TDFD) solution technique and a good agreement has been achieved.

Keywords: Skin depth; Finite Difference Time Domain (FDTD); Nonuniform Transmission Line; Time-Domain to Frequency-Domain (TDFD)

1. Introduction

With the increasing flow of data in telecommunication world and the increasing demand for higher speeds of data processing in information processing system has caused a renewed interest in modeling various nonideal effects of the conductors that interconnect the various subsystem. One of the nonideal aspects that was neglectable in slower speed systems is the skin effect impedance of the interconnect conductors of the system. This is manifested as an increase in the resistance of those conductors as the square root of frequency as the current crowds closer to the conductor surfaces [2]. The full wave analysis of transmission lines would generally provide more accuracy, but the computational expense of this method is considerably greater than the solution of the multiconductor transmission line (MTL) equations [3]. The assumption of the MTL model is that the fields satisfy a transverse electromagnetic (TEM) field structure in which the electric and magnetic fields lie in a plane that is transverse to the line axis [1].

The frequency domain analysis of MTL equation is a straightforward computational task whether the electrodes of device are considered lossy or lossless [3]. The time domain analysis of MTL equation is also simple if the lines are considered lossless. The time domain analysis of lossy MTL is significantly more difficult for several reasons. A primary reason is that the resistive losses of the device electrodes are due to skin effect and vary with frequency as √f. In this work, the skin effect is approximated by [6].

$$Z_i(x,s) = A(x) + B(x)\sqrt{s} \qquad (1)$$

That the internal impedance contains both resistance and internal inductance (due to the magnetic flux internal to the conductors).

The MTL equations in the frequency domain are given in [1]:

$$\begin{aligned} &\frac{\partial}{\partial z}\vec{V}(x,s) + Z_i(x,s)\,\vec{I}(x,s) + sL(x)\,\vec{I}(x,s) = 0 \\ &\frac{\partial}{\partial z}\vec{I}(x,s) + s\;C(x)\;\vec{V}(x,s) + G(x)\;\vec{V}(x,s) = 0 \end{aligned} \qquad (2)$$

Where $\vec{V}$ and $\vec{I}$ are vectors of the line voltages (with respect to the reference conductor) and line currents, respectively. The s variable is the Laplace transform variable.

The Laplace inverse transforms of $\vec{V}$ and $\vec{I}$ is denote as:

$$\begin{aligned} \vec{V}(x,s) &\Leftrightarrow \vec{V}(x,t) \\ \vec{I}(x,s) &\Leftrightarrow \vec{I}(x,t) \end{aligned} \qquad (3)$$

The matrix L (inductance), C (capacitance), Z_i (conductor internal impedance), and G (conductance) are in the per-unit-length. The position along the line is denoted as *x* and time is denoted as *t*.

In this paper, a lossy nonuniform transmission line is analyzed in the time domain using Finite-Difference Time-Domain (FDTD) technique. The results of FDTD technique is compared with the conventional time-domain to frequency-domain (TDFD) solution technique. There is a very good agreement between two methods while the CPU time of the solution by FDTD technique is very lowest than (≈%90) the solution by TDFD technique.

2. The FDTD Formulation

In order to illustrate method, we consider two-conductor nonuniform lines. The MTL equations in the time domain become:

$$\frac{\partial}{\partial z}\vec{V}(x,t)+Z_i(x,t)*\vec{I}(x,t)+L(x)\frac{\partial}{\partial t}\vec{I}(x,t)=0$$
$$\frac{\partial}{\partial z}\vec{I}(x,t)+C(x)\frac{\partial}{\partial z}\vec{V}(x,t)+G(x)\ \vec{V}(x,t)=0 \qquad (4)$$

The product of the internal impedance and the current in the Eq. (2) translated, in the time domain, to a convolution as

$$Z_i(x,s)\ I(x,s)=Z_i(x,t)*I(x,t) \qquad (5)$$

By substituting the Eq. (1) in Eq. (5) gives:

$$Z_i(x,s)\,I(x,s)=A(x)\,I(x,s)+B(x)\sqrt{s}\,I(x,s)$$

Therefore

$$L^{-1}\{Z_i(x,s)I(x,s)\}=L^{-1}\{A(x)I(x,s)+B(x)\frac{s}{\sqrt{s}}I(x,s)\}$$
$$=A(x)I(x,t)+L^{-1}\{\frac{B(x)}{\sqrt{s}}\}*\frac{\partial}{\partial t}I(x,t) \qquad (6)$$

The inverse Laplace transform $\frac{1}{\sqrt{s}}$ is [3]:

$$L^{-1}\{\frac{1}{\sqrt{s}}\}=\frac{1}{\sqrt{\pi\, t}} \qquad (7)$$

so, with substitute Eq. (7) in to Eq. (6) gives:

$$Z_i(x,t)*I(x,t)=L^{-1}\{Z_i(x,s)\,I(x,s)\}=$$
$$A(x)\,I(x,t)+\frac{B(x)}{\sqrt{\pi}}\int_0^t\frac{1}{\sqrt{u}}\frac{\partial}{\partial\,(t-u)}I(x,t-u)\,du \qquad (8)$$

Now a suitable technique must be selected to solve the lossy MTL equations. One of the best numerical techniques for approximated solution is the FDTD technique. This method is widely used in solving various kinds of electromagnetic problems, wherein lossy, nonlinear, inhomogeneous media and transient problem, can be considered. This technique seeks to approximate the derivatives in these equations with regard to the discrete solution points defined by the spatial and temporal cells.

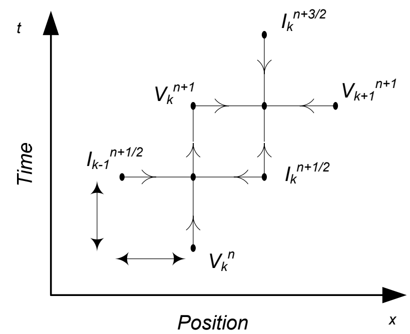

Fig. 1. The relation between the spatial and temporal discretizations to achieve second-order accuracy in the discretization of the derivations.

Generally, the accuracy of the solution depends on having sufficiently small spatial and temporal cell sizes. In order to insure stability of the discretization and to insure second-order accuracy, we interlace the N_X+1 and N_X points for voltage and current point receptivity [5]. Each voltage and adjacent current solution point is separated by Δx/2 .In addition, the time points are also interlaced, and each voltage time point and adjacent current time point are separated by Δx/2 as illustrated in Fig. (1) [3]. The solution of voltages and currents are obtained at these discrete points so we can represent an approximate solution of the MTL equations. Applying the finite difference approximation to Eq. 8 gives:

$$\int_0^t\frac{1}{\sqrt{u}}\frac{\partial}{\partial(t-u)}I(x,t-u)du=\int_0^t\frac{K(x,t-u)}{\sqrt{u}}\,du\cong$$
$$\int_0^{(n+1)\Delta t}\frac{K(x,(n+1)\Delta t-u)}{\sqrt{u}}du=\sum_{m=0}^{n}\int_{m\Delta t}^{(m+1)\Delta t}\frac{K(x,(n+1)\Delta t-u)}{\sqrt{u}}du \qquad (9)$$

We consider that the function K(x,t) is constant over the Δt segment. Therefore Eq. 9 becomes:

$$\sum_{m=0}^{n}\int_{m\,\Delta t}^{(m+1)\,\Delta t}\frac{K(x,(n+1)\Delta t-u)}{\sqrt{u}}du=$$
$$\sum_{m=0}^{n}K(x,(m+1)\Delta t-u)\int_{m\,\Delta t}^{(m+1)\,\Delta t}\frac{du}{\sqrt{u}}= \qquad (10)$$
$$\sqrt{\Delta t}\sum_{m=0}^{n}K(x,(m+1)\Delta t-u)\,P_\circ(m)$$

Where

$$P_\circ(m)=\int_m^{m+1}\frac{du}{\sqrt{u}} \qquad (11)$$

Finite difference approximation of MTL equation gives:

$$\frac{V_{k+1}^{n+1}-V_{k}^{n+1}}{\Delta x}+L(k)\frac{I_{k}^{n+3/2}-I_{k}^{n+1/2}}{\Delta t}+A(k)\frac{I_{k}^{n+3/2}+I_{k}^{n+1/2}}{2}$$

$$\frac{B(k)}{\sqrt{\pi\Delta t}}\sum_{m=0}^{n}P_{\circ}(m)\{I_{k}^{n+3/2-m}-I_{k}^{n+1/2-m}\}=0 \qquad (12a)$$

k=1, 2, …, N_X

$$\frac{I_{k}^{n+1/2}-I_{k-1}^{n+1/2}}{\Delta x}+C(k)\frac{V_{k}^{n+1}-V_{k}^{n}}{\Delta t}+ G(k)\frac{V_{k}^{n+1}+V_{k}^{n}}{\Delta t}=0 \qquad (12b)$$

k=1, 2, …, N_X

where we denote

$$V_{k}^{n}\equiv V((k-1)\Delta x, n\Delta t)$$
$$I_{k}^{n}\equiv I((k-\frac{1}{2})\Delta x, n\Delta t) \qquad (13)$$

The required recursion relations for the interior points on the line are obtained with solving the Eqs. (12a) and (12b).

$$[\frac{L(k)}{\Delta t}+\frac{A(k)}{2}+\frac{B(k)P_{\circ}(0)}{\Delta t}]I_{k}^{n+3/2}=$$
$$[\frac{L(k)}{\Delta t}-\frac{A(k)}{2}+\frac{B(k)P_{\circ}(0)}{\Delta t}]I_{k}^{n+1/2}-\frac{V_{k}^{n+1}-V_{k+1}^{n+1}}{\Delta x}$$
$$-\frac{B(k)P_{\circ}(0)}{\Delta t}\sum_{m=1}^{n}P_{\circ}(m)(I_{k}^{n-m+3/2}-I_{k}^{n-m+1/2}) \qquad (14)$$

and

$$\left[\frac{C(k)}{\Delta t}+\frac{G(k)}{2}\right]V_{k}^{n+1}=$$
$$\left[\frac{C(k)}{\Delta t}-\frac{G(k)}{2}\right]V_{k}^{n}-\frac{I_{k}^{n+1/2}-I_{k-1}^{n+1/2}}{\Delta x} \qquad (15)$$

These equations are solved in a "bootstrapping" method. The solution starts with an initially relaxed line having zero voltage and current values. First voltages along the line are solved for a fixed time from Eq. 15 in terms of the previous solutions, and then the currents are solved for using Eq. (14) in terms of these and previous values.

3. The Boundary Condition

The Eq. (15) for $k=1$ and $k=N_X+1$ become:

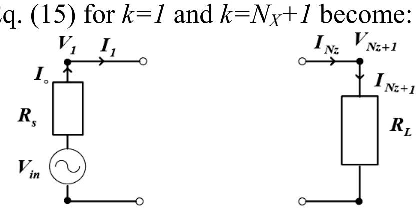

Fig. 2 .Discretization of the terminal voltages and currents.

$$\left[\frac{C(1)}{\Delta t}+\frac{G(1)}{2}\right]V_{1}^{n+1}=$$
$$\left[\frac{C(1)}{\Delta t}-\frac{G(1)}{2}\right]V_{1}^{n}-\frac{I_{1}^{n+1/2}-I_{0}^{n+1/2}}{\Delta x/2} \qquad (16)$$

and

$$\left[\frac{C(N_X+1)}{\Delta t}+\frac{G(N_X+1)}{2}\right]V_{N_X+1}^{n+1}=$$
$$\left[\frac{C(N_X+1)}{\Delta t}-\frac{G(N_X+1)}{2}\right]V_{N_X+1}^{n}-\frac{I_{N_X+1}^{n+1/2}-I_{N_X}^{n+1/2}}{\Delta x/2} \qquad (17)$$

By considering Fig. 2, this equation requires that we replace Δx with $\Delta x/2$ only for $k=1$ and $k=N_X+1$. Referring Fig. 2 we will denote the currents at the source point ($x=0$) as I_o and at the load point ($x=L$) as I_{NX+1}. By substituting this notation into Eq. (16) and Eq. (17) we obtain:

$$I_{\circ}=\frac{V_{in}^{n}+V_{in}^{n+1}-V_{1}^{n+1}-V_{1}^{n}}{2R_s} \qquad (18)$$

and

$$I_{N_X+1}=\frac{V_{N_X+1}^{n+1}+V_{N_X+1}^{n}}{2R_L} \qquad (19)$$

By substituting Eq. (18) and (19) to Eq. (16) and (17) respectively, the finite difference approximations of MTL equation become:

for $k=1$

$$V_{1}^{n+1}=[\frac{C(1)}{\Delta t}+\frac{G(1)}{2}+\frac{1}{R_s\Delta x}]^{-1}\left\{[\frac{C(1)}{\Delta t}-\frac{G(1)}{2}-\frac{1}{R_s\Delta x}]V_{1}^{n}-\frac{2}{\Delta z}I_{1}^{n+1/2}+\frac{V_{in}^{n}+V_{in}^{n+1}}{R_s\Delta x}\right\} \qquad (20)$$

for $k=2,3,\ldots,N_x$

$$V_{k}^{n+1}=\left[\frac{C(k)}{\Delta t}+\frac{G(k)}{2}\right]^{-1}\left\{\left[\frac{C(k)}{\Delta t}-\frac{G(k)}{2}\right]V_{k}^{n}-\frac{I_{k}^{n+1/2}-I_{k-1}^{n+1/2}}{\Delta x}\right\} \qquad (21)$$

for $k=N_{x+1}$

$$V_{N_x+1}^{n+1}=[\frac{C(N_X+1)}{\Delta t}+\frac{G(N_X+1)}{2}+\frac{1}{R_L\Delta x}]^{-1}\left\{[\frac{C(N_X+1)}{\Delta t}-\frac{G(N_X+1)}{2}-\frac{1}{R_L\Delta x}]V_{N_X+1}^{n}+\frac{2}{\Delta x}I_{N_X}^{n+1/2}\right\} \qquad (22)$$

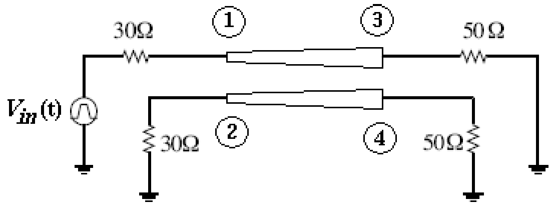

Fig. 3.Shematic of nonuniform transmission line

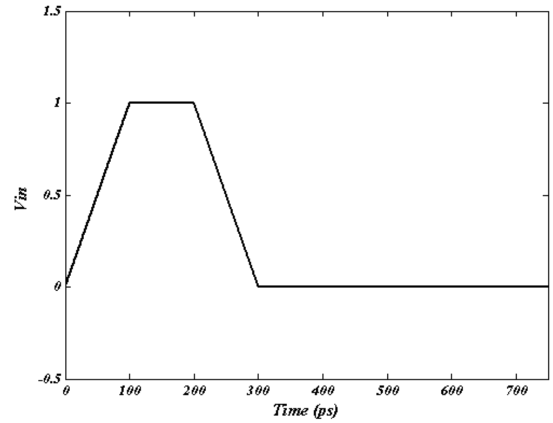

Fig.4 .The source voltage wave shape

for $k=1,2,\ldots,N_X$

$$I_k^{n+3/2} = [\frac{L(k)}{\Delta t} + \frac{A(k)}{2} + \frac{B(k)P_\circ(0)}{\Delta t}]^{-1} \Bigg\{ [\frac{L(k)}{\Delta t} - \frac{A(k)}{2} + \frac{B(k)P_\circ(0)}{\Delta t}] I_k^{n+1/2} - \frac{V_k^{n+1} - V_{k+1}^{n+1}}{\Delta x} - \frac{B(k)P_\circ(0)}{\Delta t} \sum_{m=1}^{n} P_\circ(m)(I_k^{n-m+3/2} - I_k^{n-m+1/2}) \Bigg\} \quad (23)$$

The voltages and currents are solved by iterating k for a fixed time and then iterating time.

4. Numerical Results

In order to describe the characteristics of the proposed method, we consider a two-coupled nonuniform transmission line system as shown in Fig. 3. Two nonuniform copper conductors of width $50\times(1+k(x))\ \mu m$ and thickness $5\mu m$ and conductivity 4×10^7 S/m. The input voltage source is a 1-V pulse with a 100-ps rise/fall time and a width of 100 ps. The length of the coupled line is 5cm, and the parameters are represented as follows [4]:

$$\vec{L}(x) = \begin{bmatrix} L(x) & L_m(x) \\ L_m(x) & L(x) \end{bmatrix} \quad \text{nH/m}$$

$$\vec{C}(x) = \begin{bmatrix} C(x) & C_m(x) \\ C_m(x) & C(x) \end{bmatrix} \quad \text{pF/m} \quad (24)$$

$$\vec{G}(x) = \begin{bmatrix} G(x) & 0 \\ 0 & G(x) \end{bmatrix} \quad \text{S/m}$$

where

$$L(x) = \frac{387}{1+K(x)}$$

$$L_m(x) = K(x)\,L(x) \quad (25a)$$

$$C(x) = \frac{104.3}{1-K(x)}$$

$$C_m(x) = -K(x)C(x)$$

$$G(x) = \frac{0.001}{1-K(x)} \quad (25b)$$

$$K(x) = 0.25[1+\sin(6.25\,\pi\,x + \frac{\pi}{4})]$$

The terminal conditions are

$$\vec{V}_{in} = \begin{bmatrix} 0 \\ V_s(t) \end{bmatrix},\ \vec{R}_S = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}\ \Omega \quad (26)$$

$$\vec{R}_L = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}\ \Omega$$

The per-unit-length dc resistance of each conductor is computed as [6]

$$A(x) = r_{dc} = \frac{1}{\sigma\, w(x)\; t} = \frac{100}{1+K(x)} \quad (27)$$

The factor B is computed as [6]

$$B = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{2\,(t+w(x))} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{2\times10^{-6}\times(5+50(1+K(x)))} \quad (28)$$

The transient voltages at the far end of the lines (ports 3 and 4) are depicted in Figs 5 and 6. The Figs 7 and 8 show the transient voltage at the near end of the lines (ports 1 and 2). It is assumed that the nonuniform lines are divided into 30 segments in the length and the time step is chosen to be 0.5 ps in the FDTD method. The results of FDTD technique for lossy nonuniform transmission lines are compared with the TDFD method with 500 harmonics and a very good agreement has been achieved. Also this transmission line with the lossless assumption is simulated by FDTD technique. The results of nonuniform lossless transmission line are show in Figs 5-8.

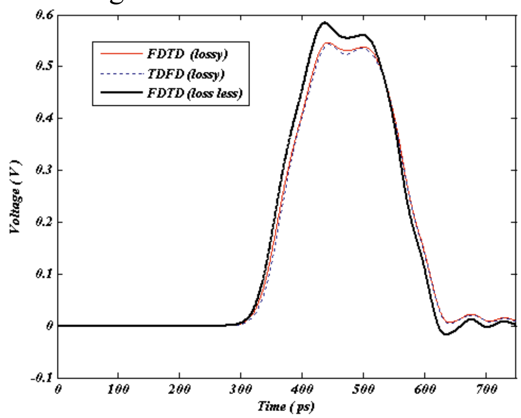

Fig. 5. the voltage wave form at the port 3 for loss less and lossy transmission line using FDTD and TDFD technique

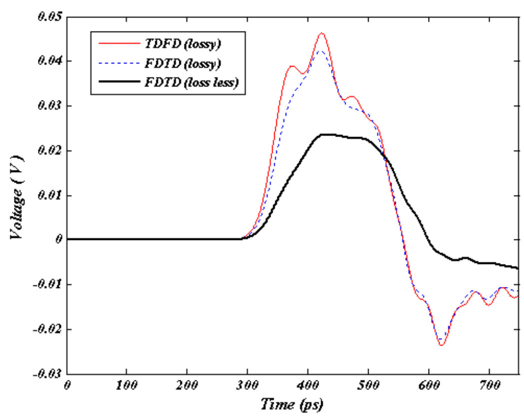

Fig. 6. the voltage wave form at the port 4 for loss less and lossy transmission line using FDTD and TDFD technique

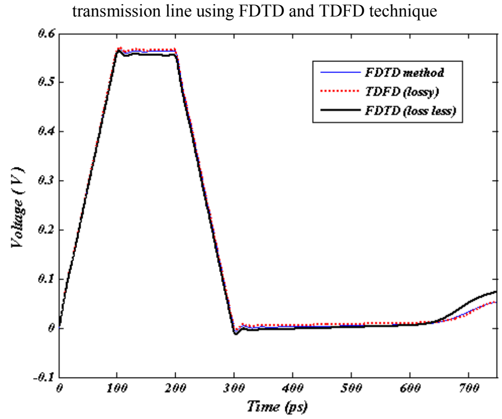

Fig. 7. the voltage wave form at the port1 for loss less and lossy transmission line using FDTD and TDFD technique

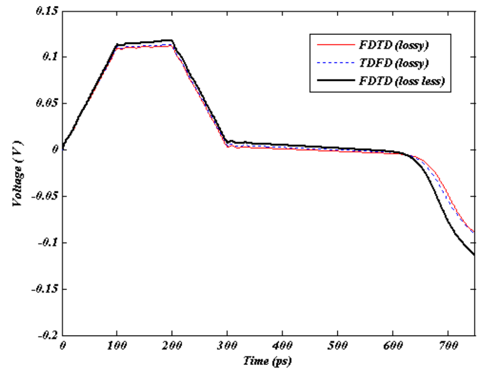

Fig. 8. the voltage wave form at the port 2 for loss less and lossy transmission line using FDTD and TDFD technique

5. Conclusion

An accurate and efficient method for transient analysis of lossy nonuniform transmission lines using the FDTD technique has been described. The results of this method are confirmed with the TDFD method. The results show that at the high frequency application, the frequency dependent losses should take account in the analysis of the transmission line. The CPU time of the FDTD is %90 less than the TDFD method.

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